REMARKS ON PROFESSOR LYLE'S POSTULATE I. OF EUCLID'S ELEMENTS.

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Professor Lyle, in No. 1 of THE AMERICAN MATHEMATICAL MONTHLY, falls into an error, through misapprehending the meaning of Lobatschewsky.

According to Lobatschewsky the angle-sum of a rectilineal triangle decreases as the area of the triangle increases, but is always less than two right angles.

Lobatschewsky' sgeometry does not apply to the plane, nor to space as we know it, but to what has since been termed a pseudospherical surface,or one of uniform negative curvature in the same sense that the surface of a sphere is of uniform positive curvature. Such a surface cannot be fully constructed,and the theorems of Lobatschewsky are seemingly impossible; but his geometry is consistant with itself and contradicts none of the postulates or axioms of Euclid except the 12th. His straight line is not, (it is true,) really straight, but is the shortest distance between two points, and lying wholly in the given space. A straight line may be drawn between any two points in the space, and a triangle can be formed of three straight lines joining any three points.

This being premised, the Professor's first error is in defining a finite straight line as one that has two ends, and in confounding "infinite" and "boundless". He may refer to a *terminated* straight line, and his definition is then correct.

Now, it is true, a straight line can be drawn from any point in AC to any point in CB, and the triangle ECF will have an angle-sum greater than two right angles$-a$. This however, is not contrary to the hypothesis that the angle-sum shall be less than two right angles. No matter how small a is taken it can still be divided. Though the angle C be as nearly equal to two right angles as you choose yet E and F, taken together, will not entirely make up the difference. If a is taken infinitely small the area of the triangle ECF will be infinitely small, and its angle-sum will differ from two right angles by less than any assignable quantity.

MORE REMARKS ON DIVISION.

By J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

An introduction of half a hundred lines from ancient history, overshadowing a mere assertion as a conclusion, might give these remarks an air of profundity, but that species of pedantry, being an "idol" which neither Gauss nor Argand has yet knocked down, is a pet whose sacred form profane hands must not pollute.

Mr. Smith says that multiplication was, originally, a mere process of adding; but what is mathematically true today was true from the beginning and always will be true. It is thus seen that Mr. Smith either corroborates my statement as to multiplication, or denies the fixed unchangeable character of mathematical truth.

When children are learning multiplication of integers in arithmetic they *do* see, they *can* see, they are *asked* to see but one explanation, viz., that multiplication is a process of adding. The same language will not explain multiplication by a fraction.

In *general*, multiplication is finding one of the four terms of a proportion; *i. e.*, it is finding a number that bears the same relation to a multiplicand as the multiplier, or operator, bears to unity. The *arithmetical* conception of multiplication by integers admits but one definition of the process, which is that almost universally given in the text-books—a process of adding. The arithmetical conception of discrete number does not enable us to locate $\sqrt{3}$, although it is a real number between 1 and 2. To divide 10 into two parts whose products shall be 40 is impossible by arithmetic. That is to say, what is legitimate and possible in an algebraic or geometric sense may be impossible in an arithmetical sense.

In the domain of pure arithmetic $\sqrt{-15}$ is no more absurd than $\$12 \div 2$.

A *concrete number* in arithmetic is, in the child's mind, a number of objects, as 8 books. These may be divided *physically* into 2 equal parts, but 8 books$\div 2$ as an arithmetical operation is absurd. We may divide x^2 by x, but not by y. The operation may be indicated $\left(\frac{x^2}{y}\right)$ but can not be *performed.* Ten 5's can not be divided by five 10's unless both be reduced to the same unit.

Multiplication is *one* thing only, and division, its inverse, can be but one thing. If division is a process of finding how often one quantity is contained in another, it cannot at the same time be a process of finding one of the equal parts of a quantity. The latter is an application of division.

In a recent issue of the *Popular Educator* Dr. McLellan, author of "Applied Psychology," etc., devotes several columns to proving that a concrete number can be divided by an abstract. His whole argument is based upon the commutative law of multiplication, which Mr. Smith has consigned to the "museum of antiquities"! Without the "old idol," the commutative law, it is not possible to prove that $12 can be divided by 2, in an arithmetical sense.

While Mr. Smith would appear as an exponent of progress, he is championing an "old fogy" notion. Before he was born arithmetics taught that $\$12 \div 2 = \$6$, but recent authors have advanced a step, and, strange as it may seem, he dons his knightly armor to do battle with the progressive idea. Since the commutative law of multiplication has been duly labeled and placed in the "museum," does Mr. Smith intend to "do or die" in defence of its relative—the commutative law of division ?

We find that $\frac{1}{2}$ of $\$12 = \$6$. It will be noticed that the 12 has been divid-

ed, the \$ has *not*. The \$ has not entered into the division at all. It is simply annexed to the 6. The principle is evident.

ARE DIFFERENTIALS FINITE QUANTITIES?

By JOHN N. LYLE, Ph. D., Professor of Mathematics, Westminster College, Fulton, Missouri.

In seeking for a correct answer to the above question let us reconsider in detail a familiar elementary example.

Let $u=x^2\ldots(1)$, in which u is a function of the independent variable x that increases in value *uniformly*.

The *increment* of the variable x in *a unit of time* is the *rate of variation* of the variable x and may be appropriately represented by the symbol dx.

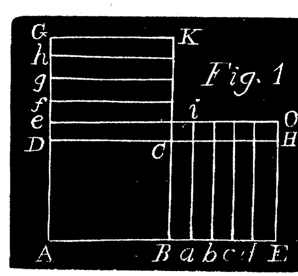
When the variable x reaches the value x' or AB, Fig. 1, and the function u, the corresponding value u' or AC we shall have $u'=x'^2\ldots.(2)$.

Let $\Delta x=\dfrac{dx}{n}$, in which Δx represents the increment of the variable x in $\dfrac{1}{n}$ of a unit of time. Since x varies uniformly, $n\times\Delta x$ will equal dx; that is, will equal the rate of variation of the independent variable x.

When the variable x reaches the value x''; that is, $x'+\dfrac{dx}{n}$, or Aa, and the function u, the corresponding value u'' or Ai we shall have $u''=x''^2=\left(x'+\dfrac{dx}{n}\right)^2\ldots.(3)$.

Subtract (2) from (3).

Then $u''-u'$, or $\Delta u'=x''^2-x'^2=2x'\dfrac{dx}{n}+\dfrac{dx^2}{n^2}\ldots.(4)$.



Multiply both members by n. Then $n\times\Delta u'=2x'dx+\dfrac{dx^2}{n}\ldots.(5)$. The second member of (5) is made up of two parts, one of which, $2x'dx$, is constant whatever the value of n may be, and the other, $\dfrac{dx^2}{n}$, decreases without limit as n increases without limit. $\dfrac{dx^2}{n}$ is n times $\dfrac{dx^2}{n^2}$, the addition to the increment of the function in $\dfrac{1}{n}$ of the unit of time due to the *increase* in the tendency of the function to vary after passing the value u'.

$\dfrac{2x'dx}{n}\times n$, that is, $2x'dx$ is the increment of the function u in the unit of time that is due to its tendency to vary when it reaches the value u'.

The increment $2x'dx$ is received *uniformly* during the unit of time